I Shape Beam-Column Per AISC 360-10

Ref: AISC 14th Edition

Ver A.1
Build 15.11.09

INPUT

$I_memb := \text{"W10X22"}$ I Member Shape

$L_{bx} := 10 \cdot ft$ Unbraced Compressive Length x axis

$P_{axial} := \begin{pmatrix} -10.0 \\ 0.0 \end{pmatrix} \cdot kip$ Envelope of Axial Loads (Negative for Compression) (Positive for Tension)

$L_{by} := 10 \cdot ft$ Unbraced Compressive Length y Axis (Minor Axis)

$L_{bz} := L_{by}$ Unbraced Torsional Length

$M_x := 35 \cdot kip \cdot ft$ Bending About X (Major) Axis

$M_y := 1.0 \cdot kip \cdot ft$ Bending About Y (Minor) Axis

$K_x := 1.0$ $K_y := 1.0$ $K_z := 1.0$

$L_b := L_{by}$ Lateral Unbraced Length of Beam

$C_b := 1.0$ $F_y := 50 \cdot ksi$

Global Unit Definitions:

Seed Rev 1.0a

$AISC_I_Shape_TBL :=$

	0	1
0	"W"	"W44X335"
1	"W"	...

From AISC Shapes Database v14.1.xls

Extra Line of I Shape properties for "Size Not Found"

$X_{extra.line_{0,78}} := 0$ $I_Prop := stack(AISC_I_Shape_TBL, X_{extra.line})$

$I_Prop =$

	0	1
0	"W"	"W44X335"
1	"W"	"W44X290"
2	"W"	"W44X262"
3	"W"	"W44X230"
4	"W"	...

Index Searching Function

```
search_indx(Shape) := | i ← 0
                      | while  i < rows(I_Prop) − 1
                      |     | break  if  I_Prop_{i,2} = Shape
                      |     | i ← i + 1
                      |     | continue
                      | i
```

Description of Column Values in I_Prop Table

2	4	6	13	10	14	18	20	6	34	36	35	37	38	40	39	41
Shape	wgt_{ft}	d	t_w	b_f	t_f	k_{des}	k_1	A	I_x	S_x	Z_x	r_x	I_y	S_y	Z_y	r_y

$i_b := search_{indx}(I_memb)$ $\quad i_b = 245.000$ $\quad I_Prop_{i_b,2} = \text{"W10X22"}$

$Imemb := I_Prop_{i_b,2} = \text{"W10X22"}$ Member Size Designation

$A_g := I_Prop_{i_b,5} \cdot in^2 = 6.490 \cdot in^2$ Cross Section Area of I Shape

$d := I_Prop_{i_b,6} \cdot in = 10.200 \cdot in$ T Depth

$b_f := I_Prop_{i_b,11} \cdot in = 5.750 \cdot in$ T Flange Width

$t_w := I_Prop_{i_b,16} \cdot in = 0.240 \cdot in$ T Stem Thickness

$t_f := I_Prop_{i_b,19} \cdot in = 0.360 \cdot in$ T Flange Thickness

$k_{des} := I_Prop_{i_b,24} \cdot in = 0.660 \cdot in$ k design distance

$S_x := I_Prop_{i_b,40} \cdot in^3 = 23.200 \cdot in^3$ Section Modulus (to tip of T Stem)

$S_y := I_Prop_{i_b,44} \cdot in^3 = 3.970 \cdot in^3$ Section Modulus About Plane of Symmetry

$I_x := I_Prop_{i_b,38} \cdot in^4 = 118.000 \cdot in^4$ Moment of Inertia about x axis

$I_y := I_Prop_{i_b,42} \cdot in^4 = 11.400 \cdot in^4$ Moment of Inertia about y axis

$J := I_Prop_{i_b,49} \cdot in^4 = 0.239 \cdot in^4$ Polor Moment of Inertia

$r_x := I_Prop_{i_b,41} \cdot in = 4.270 \cdot in$ Radius of Gyration About x axis

$r_y := I_Prop_{i_b,45} \cdot in = 1.330 \cdot in$ Radius of Gyration About y axis

$Z_x := I_Prop_{i_b,39} \cdot in^3 = 26.000 \cdot in^3$ Plastic Section Modulus About x axis

$Z_y := I_Prop_{i_b,43} \cdot in^3 = 6.100 \cdot in^3$

$C_w := I_Prop_{i_b,50} \cdot in^6 = 275.000 \cdot in^6$ Warping Constant

$r_{ts} := I_Prop_{i_b,75} \cdot in = 1.550 \cdot in$ $\quad h_o := I_Prop_{i_b,76} \cdot in = 9.840 \cdot in$

I Shape Properties

(From AISC Database v14.1)

Imemb = "W10X22" I Size ("0.000" if section does not exist)

$d = 10.200 \cdot in$	$S_x = 23.200 \cdot in^3$	$I_x = 118.00 \cdot in^4$	$r_x = 4.270 \cdot in$	$k_{des} = 0.660 \cdot in$
$t_w = 0.240 \cdot in$	$S_y = 3.970 \cdot in^3$	$I_y = 11.40 \cdot in^4$	$r_y = 1.330 \cdot in$	$C_w = 275 \cdot in^6$
$b_f = 5.750 \cdot in$	$Z_x = 26.000 \cdot in^3$	$h_o = 9.840 \cdot in$	$r_{ts} = 1.550 \cdot in$	
$t_f = 0.360 \cdot in$	$Z_y = 6.100 \cdot in^3$	$A_g = 6.490 \cdot in^2$	$J = 0.239 \cdot in^4$	

$\Omega_{1.67} := 1.67$ $E := 29000 \cdot ksi$ $G := 11200 \cdot ksi$ $h := d - 2 \cdot k_{des} = 8.880 \cdot in$

Check Axial Compression Capacity of I Shape

Check Slenderness of I Shape Flange and Web Subject to Axial Compression

$\lambda_f := \frac{b_f}{2 \cdot t_f} = 7.986$ Slenderness of Flanges

$\lambda_{rf} := 0.56 \cdot \sqrt{\frac{E}{F_y}} = 13.487$ Slenderness Limit of Flanges

$\lambda_w := \frac{h}{t_w} = 37.000$ Slenderness of I Shape Web

$\lambda_{rw} := 1.49 \cdot \sqrt{\frac{E}{F_y}} = 35.884$ Slenderness Limit of I Shape Web

$$I_Shape_has := \begin{cases} \text{"Slender Web and Flanges"} & \text{if } (\lambda_f \ge \lambda_{rf}) \wedge (\lambda_w \ge \lambda_{rw}) \\ \text{"Slender Flanges"} & \text{if } (\lambda_f \ge \lambda_{rf}) \wedge (\lambda_w < \lambda_{rw}) \\ \text{"Slender Web"} & \text{if } (\lambda_f < \lambda_{rf}) \wedge (\lambda_w \ge \lambda_{rw}) \\ \text{"no Slender Elements"} & \text{if } (\lambda_f < \lambda_{rf}) \wedge (\lambda_w < \lambda_{rw}) \end{cases}$$

I_Shape_has = "Slender Web"

Calculate deduction for member with slender elements (as required)

$f := F_y$ Per AISC on pg 16.1-43 user note f may conservatively be taken as equal to Fy

$$h_e := \min\left[1.92 \cdot t_w \cdot \sqrt{\frac{E}{f}} \cdot \left(1 - \frac{0.34}{\frac{h}{t_w}} \cdot \sqrt{\frac{E}{f}}\right), h\right] = 8.642 \cdot in$$

$h_{deduct} := h - h_e = 0.238 \cdot in$

$A_e := A_g - h_{deduct} \cdot t_w = 6.433 \cdot in^2$ $\qquad Q_a := \frac{A_e}{A_g} = 0.991$

$Q_s := 1.0$ For rolled I shape members say Qs = 1.0

$Q := Q_s \cdot Q_a = 0.991$

Calculate Critical Compressive Stresses for both Primary Axes of the Member

x axis elastic buckling stress

$$\frac{K_x \cdot L_{bx}}{r_x} = 28.1$$

$$F_{ex} := \frac{\pi^2 \cdot E}{\left(\frac{K_x \cdot L_{bx}}{r_x}\right)^2} = 362.40 \cdot ksi$$

y axis elastic buckling stress

$$\frac{K_y \cdot L_{by}}{r_y} = 90.2$$

$$F_{ey} := \frac{\pi^2 \cdot E}{\left(\frac{K_y \cdot L_{by}}{r_y}\right)^2} = 35.16 \cdot ksi$$

For I Shaped Members

$$F_{crx} := \begin{cases} Q \cdot \left(0.658^{\frac{Q \cdot F_y}{F_{ex}}}\right) \cdot F_y & \text{if } \frac{K_x \cdot L_{bx}}{r_x} \le 4.71 \cdot \sqrt{\frac{E}{Q \cdot F_y}} \\ \left(0.877 \cdot F_{ex}\right) & \text{if } \frac{K_x \cdot L_{bx}}{r_x} > 4.71 \cdot \sqrt{\frac{E}{Q \cdot F_y}} \end{cases} = 46.802 \cdot ksi$$

$$F_{cry} := \begin{cases} Q \cdot \left(0.658^{\frac{Q \cdot F_y}{F_{ey}}}\right) \cdot F_y & \text{if } \frac{K_y \cdot L_{by}}{r_y} \le 4.71 \cdot \sqrt{\frac{E}{Q \cdot F_y}} \\ \left(0.877 \cdot F_{ey}\right) & \text{if } \frac{K_y \cdot L_{by}}{r_y} > 4.71 \cdot \sqrt{\frac{E}{F_y}} \end{cases} = 27.473 \cdot ksi$$

Calculate the torsional elastic buckling stess (Eq. E4-4)

$$F_{e.flex.tor} := \left[\frac{\pi^2 \cdot E \cdot C_w}{\left(K_z \cdot L_{bz}\right)^2} + G \cdot J\right] \cdot \frac{1}{I_x + I_y} = 62.927 \cdot ksi$$

$$F_{cr.flex.tor} := \begin{cases} \left(0.658^{\frac{F_y}{F_{e.flex.tor}}}\right) \cdot F_y & \text{if } \frac{F_y}{F_{e.flex.tor}} \le 2.25 \\ \left(0.877 \cdot F_{e.flex.tor}\right) & \text{if } \frac{F_y}{F_{e.flex.tor}} > 2.25 \end{cases} = 35.854 \cdot ksi$$

Determine Controling Critical Compressive Stress

$$F_{cr.comp} := \min\left(F_{crx}, F_{cry}, F_{cr.flex.tor}\right) = 27.473 \cdot ksi$$

Nominal Axial Capacity in Compression

$$P_{n.comp} := F_{cr.comp} \cdot A_g = 178.298 \cdot kip$$

Allowable Axial Compressive Stress

$$F_{ca.comp} := \frac{F_{cr.comp}}{\Omega_{1.67}} = 16.451 \cdot ksi$$

Available Axial Compressive Strength

$$P_{ca.comp} := \frac{P_{n.comp}}{\Omega_{1.67}} = 106.765 \cdot kip$$

Check Axial Tensile Capacity of I Shape

Nominal Axial Capacity in Tension

$$P_{n.ten} := F_y \cdot A_g = 324.500 \cdot kip$$

Available Stress in Tension

$$F_{ca.ten} := \frac{P_{n.ten}}{\Omega_{1.67} \cdot A_g} = 29.940 \cdot ksi$$

Check Bending Capacity of I Shape

AISC Section F2.1 Yielding & F3.2 Flange Local Buckling of I Shapes

$$M_p := F_y \cdot Z_x = 108.333 \cdot ft \cdot kip \qquad \lambda_f = 7.986$$

$$\lambda_{pfb} := 0.38 \cdot \sqrt{\frac{E}{F_y}} = 9.152$$

Local Buckling Compactness Limit of Flange in Bending

$$\lambda_{rfb} := 1.0 \cdot \sqrt{\frac{E}{F_y}} = 24.083$$

Local Buckling Slenderness Limit of Flange in Bending

$$I_Shape_has := \begin{cases} \text{"Compact Flanges for Bending"} & \text{if } \left(\lambda_f \le \lambda_{pfb}\right) \\ \text{"Non Compact Flanges for Bending"} & \text{if } \lambda_{pfb} < \lambda_f \le \lambda_{rfb} \\ \text{"Slender Flanges in Bending - Worksheed does not Apply"} & \text{if } \lambda_f > \lambda_{rfb} \end{cases}$$

I_Shape_has = "Compact Flanges for Bending"

$$M_{n.bx} := \min\left[M_p, M_p - \left(M_p - 0.7 \cdot F_y \cdot S_x\right) \cdot \frac{\lambda_f - \lambda_{pfb}}{\lambda_{rfb} - \lambda_{pfb}}\right] = 108.333 \cdot kip \cdot ft$$

Nominal Flexural Strength of Bending About X (Major) Axis for Beam With Full Lateral Bracing

AISC Section F2.2 Lateral Torsional Buckling

$$L_p := 1.76 \cdot r_y \cdot \sqrt{\frac{E}{F_y}} = 4.698 \cdot ft \qquad c := 1.0$$

$$L_r := 1.95 \cdot r_{ts} \cdot \frac{E}{0.7 \cdot F_y} \cdot \sqrt{\frac{J \cdot c}{S_x \cdot h_o} + \sqrt{\left[\left(\frac{J \cdot c}{S_x \cdot h_o}\right)^2 + 6.76 \cdot \left(\frac{0.7 \cdot F_y}{E}\right)^2\right]}} = 13.772 \cdot ft$$

$$F_{cr.b} := \frac{C_b \cdot \pi^2 \cdot E}{\left(\frac{L_b}{r_{ts}}\right)^2} \cdot \sqrt{1 + 0.078 \cdot \frac{J \cdot c}{S_x \cdot h_o} \cdot \left(\frac{L_b}{r_{ts}}\right)^2} = 58.279 \cdot ksi$$

$$M_{n.lat.tor} := \begin{cases} M_p & \text{if } L_b \le L_p \\ \min\left[C_b \cdot \left[M_p - \left(M_p - 0.7 \cdot F_y \cdot S_x\right) \cdot \left(\frac{L_b - L_p}{L_r - L_p}\right)\right], M_p\right] & \text{if } L_p < L_b \le L_r \\ \min\left(F_{cr.b} \cdot S_x, M_p\right) & \text{if } L_b > L_r \end{cases} = 84.572 \cdot ft \cdot kip$$

$$M_{n.x} := \min\left(M_{n.bx}, M_{n.lat.tor}\right) \qquad M_{n.x} = 84.572 \cdot kip \cdot ft$$

Nominal Flexural Strength of Beam Bending About X (Major) Axis

$$M_{py} := \min\left(F_y \cdot Z_y, 1.6 \cdot F_y \cdot S_y\right) = 25.417 \cdot kip \cdot ft$$

Plastic Flexural Strength of Beam Bending About Y Axis

$$M_{n.y} := \min\left[M_{py}, M_{py} - \left(M_{py} - 0.7 \cdot F_y \cdot S_y\right) \cdot \frac{\lambda_f - \lambda_{pfb}}{\lambda_{rfb} - \lambda_{pfb}}\right] = 25.417 \cdot kip \cdot ft$$

Nominal Flexural Strength of Beam Bending About Y Axis

$$i := 0 .. 1 \qquad P_r := P_{axial} \qquad M_{rx} := M_x \qquad M_{ry} := M_y$$

Perform Unity Check

Per AISC H1-1a or H1-1b as appropriate

$$P_{c_i} := \begin{cases} \dfrac{P_{n.ten}}{\Omega_{1.67}} & \text{if } P_{axial_i} \geq 0 \\ \dfrac{-P_{n.comp}}{\Omega_{1.67}} & \text{if } P_{axial_i} < 0 \end{cases}$$

$$P_c = \begin{pmatrix} -106.8 \\ 194.3 \end{pmatrix} \cdot kip$$

Design (Available) Strengths
(positive for tensile)
(negative for compressive)

$$\frac{P_r}{P_c} = \begin{pmatrix} 0.094 \\ 0.000 \end{pmatrix}$$

$$M_{cx} := \frac{M_{n.x}}{\Omega_{1.67}} = 50.6 \cdot kip \cdot ft$$

$$M_{cy} := \frac{M_{n.y}}{\Omega_{1.67}} = 15.2 \cdot kip \cdot ft$$

Design (Available)
Bending Strengths

$$U_i := \begin{cases} \dfrac{P_{r_i}}{P_{c_i}} + \dfrac{8}{9} \cdot \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}} \right) & \text{if } \dfrac{P_{r_i}}{P_{c_i}} \geq 0.2 \\ \dfrac{P_{r_i}}{2 \cdot P_{c_i}} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}} \right) & \text{if } \dfrac{P_{r_i}}{P_{c_i}} < 0.2 \end{cases}$$

H1-1a

H1-1b

$$P_r = \begin{pmatrix} -10.000 \\ 0.000 \end{pmatrix} \cdot kip$$

$$M_{rx} = 35.000 \cdot kip \cdot ft$$

$$M_{ry} = 1.000 \cdot kip \cdot ft$$

Required Strengths
are Equal to
the Applied Loads

$$IShape := \begin{cases} \text{"Fails Unity Check"} & \text{if } \max(U) > 1 \\ \text{"Passes Unity Check"} & \text{if } \max(U) \leq 1 \end{cases}$$

$$U = \begin{pmatrix} 0.804 \\ 0.757 \end{pmatrix}$$

$$\max(U) = 0.804$$

IShape = "Passes Unity Check"

Make Unity Check Using Alternative Equation H2-1

Calculate required stress resisistance
leveles based upon applied loads

$f_{ra} := \frac{P_{axial}}{A_g} = \begin{pmatrix} -1.541 \\ 0.000 \end{pmatrix} \cdot ksi$ $\quad f_{rbx} := \frac{M_x}{S_x} = 18.103 \cdot ksi$ $\quad f_{rby} := \frac{M_y}{S_y} = 3.023 \cdot ksi$

$F_{cbx} := \frac{M_{n.x}}{\Omega_{1.67} \cdot S_x} = 26.194 \cdot ksi$ Allowable Stress in Bending About the X (Major) Axis

$F_{cby} := \frac{M_{n.y}}{\Omega_{1.67} \cdot S_y} = 46.004 \cdot ksi$ Allowable Stress in Bending About the X (Major) Axis

$F_{ca_i} := \begin{cases} F_{ca.ten} & \text{if } P_{axial_i} \geq 0 \\ -F_{ca.comp} & \text{if } P_{axial_i} < 0 \end{cases}$ $\quad F_{ca} = \begin{pmatrix} -16.451 \\ 29.940 \end{pmatrix} \cdot ksi$ Design (Allowable) Axial Stresses

AISC Eq H2-1 (Alternative equations Eq H1-1a or H1.1b typically provide more capacity)

$U_i := \frac{f_{ra_i}}{F_{ca_i}} + \frac{f_{rbx}}{F_{cbx}} + \frac{f_{rby}}{F_{cby}}$ $\quad U = \begin{pmatrix} 0.850 \\ 0.757 \end{pmatrix}$ Unity Check Based Upon Equartion H2-1

$IShape := \begin{cases} \text{"Fails H2-1 Unity Check"} & \text{if } \max(U) > 1 \\ \text{"Passes H2-1 Unity Check"} & \text{if } \max(U) \leq 1 \end{cases}$

$\max(U) = 0.850$

$IShape = \text{"Passes H2-1 Unity Check"}$